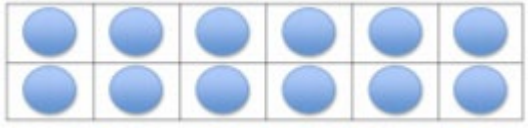
Woodland Academy Trust

Year 4 Calculation Document

Updated September 2021

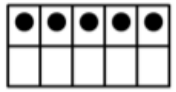
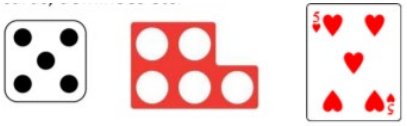
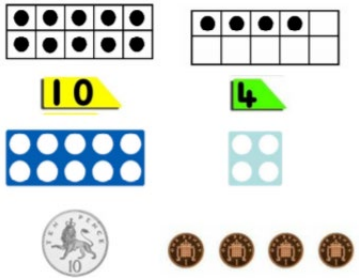
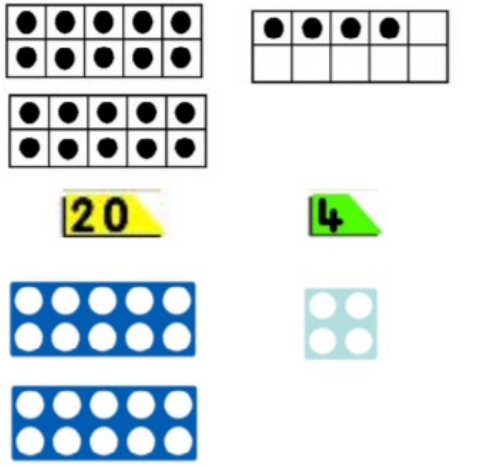
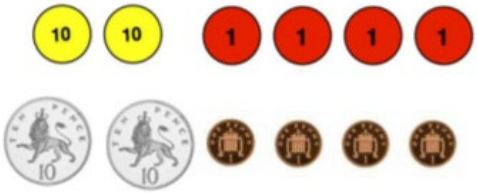
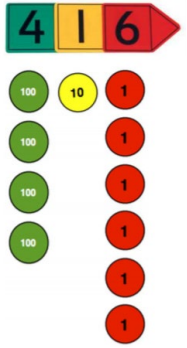
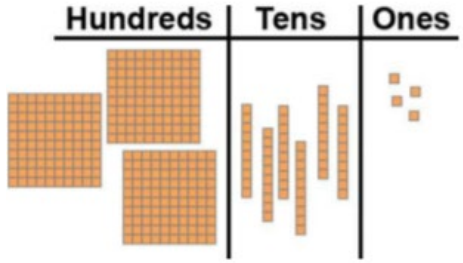
Progression in the use of manipulatives to support learning (How we support children's concrete understanding of maths)						
Foundation	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Real-life objects	Real-life objects	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards	Mini-whiteboards
0 – 9 digit cards	0 – 9 digit cards	Place value cards			Protractors	Protractors
Number track/line to 20	Number line to 20 and 50	Number line to 100	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick	Meter/Counting stick
		Transparent rulers	Transparent rulers	Transparent rulers	Transparent rulers	Transparent rulers
Tens frame	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square	Tens frame and hundred square
Building blocks	Place value charts – Tens and ones	Place value charts – Ones to hundreds	Place value charts – Ones to Thousands	Place value charts – Ones to Ten thousands	Place value charts to a million and three decimal places	Place value charts to 10 million and three decimal places
Containers that are different shapes and sizes	Containers that are different shapes and sizes	Fraction bars, walls, circles (centralised storage)				
Numicon shapes	Numicon shapes/ Dienes	Dienes	Dienes	Dienes	Dienes	Dienes
Sorting hoops	Sorting hoops	Sorting hoops	Place value counters	Place value counters	Place value counters	Place value counters
Big Dice	Place value arrow cards – tens and ones	Place value arrow cards – tens and ones	Place value arrow cards – H, T, O	Place value arrow cards – H, T, O	Place value arrow cards	Place value arrow cards
Part-part-whole mat	Part-part-whole mat	Part-part-whole mat	Part-part-whole model	Part-part-whole model	Part-part-whole model	Part-part-whole model
Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters	Transparent counters
Bar model with real-life objects	Bar model pictorial objects/ representative objects e.g. counters	Bar model with counters /Dienes progressing to numbers	Plastic mirrors	Plastic mirrors	Plastic mirrors	Plastic mirrors
Bead strings – ten	Bead strings – twenty/fifty	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred	Bead strings - hundred
Dice	Dice	Dice	Dice	Dice	Dice	Dice
Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods	Cuisenaire rods
Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters	Double sided counters
Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount	Multilink – use one colour to model an amount
Maths balances			Weighing scales			
Solid geometric shapes (centralised storage)						
Coins and notes (centralised storage)						
Clock (geared) (centralised storage)						

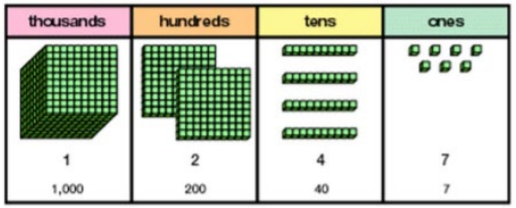
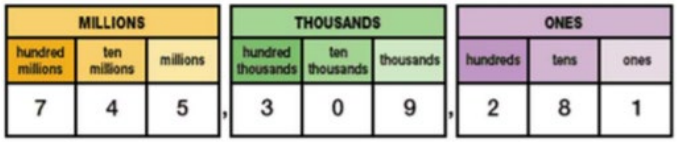
Maths Working Wall (How we use displays to support children's understanding of mathematical concepts)		
Build it	Use a real-life representation of the concept, which children can see, touch and feel.	
Draw it	Show a pictorial representation of the concept.	
Solve it	Show the mathematical representation of the concept	6 x 2 = 12 2 x 6 = 12 12 ÷ 2 = 6 12 ÷ 6 = 2 Factors of 12 are: 1, 2, 3, 4, 6 and 12
Practise it	Encourage children to practice the concept. Interactive opportunity – ask children to respond to questions, encourage them to add what they know, leave homework for children to take to master the concept.	1 x 2 = 2 2 x 2 = 4 3 x 2 = 6 etc.
Challenge it	Set a challenge to be solved. Interactive opportunity – leave real-life objects or manipulatives for children to use to help solve the challenge.	How many different ways can 12 eggs be arranged into arrays? What if you try 24 eggs?
Say it	Use vocabulary related to the concept	Multiply, multiplication , repeated addition, array, divide, group, multiples, factors

Classroom visual prompts (How we represent maths to the children pictorially)						
Foundation	**Year 1**	**Year 2**	**Year 3**	**Year 4**	**Year 5**	**Year 6**
Big focus 10	Big focus 20	Big focus 100				
Place Value Chart 10	Place Value Chart 20	Place Value Chart 100	Place Value Chart Th- tenths	Place Value Chart Tth- Hundredths	Place Value Chart M- Thousandths	Place Value Chart M- Thousandths
Numicon number line with Numicon shapes	Numicon number line with Numicon shapes	Fractions number line	Fractions number line	Fractions and decimals number line	Fractions, decimals and percentages number line	Fractions, decimals and percentages number line
Odd and even numbers	Odd and even numbers	Odd and even numbers	Factors and multiples	Factors and multiples	Factors, prime and composite numbers	Number properties
	Number bonds to 10 Number bonds to 20	Number bonds to 10 Multiples of 10 totalling 100	Number bonds to 10 Multiples of 10 totalling 100			
0 – 20 number line / track	0 -50 number line	0 – 100 number line	Number line to 100	Number line including negative numbers	Number line including negative numbers	Number line including negative numbers
	100 square	100 square	100 square	100 square	100 square	100 square
Number names from 0 - 10	Number names of multiples of 10	Number names from 0 – 100	Number names from 0 - 1000	Number names to hundred thousands	Number names to one million	Number names to million
Real coins and Large coins	Real coins and Large coins	Real coins and Large coins	Real coins and Large coins	Real coins and Large coins	Real coins and Large coins	Real coins and Large coins
Counting in 1s and 2s	2, 5 and 10 multiplication tables	2, 4 and 8 multiplication tables	3, 6 and 12 multiplication tables	7, 9 and 11 multiplication tables All multiplication tables up to 12 x 12	All multiplication tables up to 12 x 12	All multiplication tables up to 12 x 12
Counting in 1s and 2s multiplication table patterns and divisibility rules and connections.	2, 5 and 10 multiplication table patterns and divisibility rules and connections. Display after introducing the times tables to the children.	2, 4 and 8 multiplication table patterns and divisibility rules and connections. Display after introducing the times tables to the children.	3, 6 and 12 multiplication table patterns and divisibility rules and connections. Display after introducing the times tables to the children.	All multiplication table patterns and divisibility rules Connections between 5/10, 3/6/12, 2/4/8 Also focus on 1, 7, 9 and 0 multiplication table.	All multiplication table patterns and divisibility rules Connections between 5/10, 3/6/12, 2/4/8 Also focus on 1, 7, 9 and 0 multiplication table. Square and cube numbers	All multiplication table patterns and divisibility rules Connections between 5/10, 3/6/12, 2/4/8 Also focus on 1, 7, 9 and 0 multiplication table. Square and cube numbers
			Roman numerals	Roman numerals	Roman numerals	Roman numerals
The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to	The = sign means not an answer but is equivalent to
2D and 3D shapes	2D and 3D shapes	2D and 3D shapes	2D and 3D shapes	2D and 3D shapes	2D and 3D shapes	2D and 3D shapes

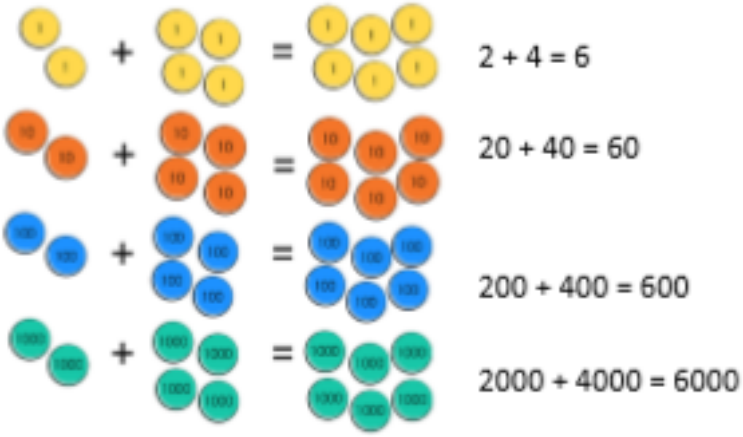
	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on- using cubes. Regrouping to make 10 using ten frame.	Adding three single digits. Use of base 10 to combine two numbers.	Column method- regrouping. Using place value counters (up to 3 digits).	Column method- regrouping. (up to 4 digits)	Column method- regrouping. Use of place value counters for adding decimals.	Column method- regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10 using the ten frame	Counting back Find the difference Part whole model Make 10 Use of base 10	Column method with regrouping. (up to 3 digits using place value counters)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. Abstract for whole numbers. Start with place value counters for decimals- with the same amount of decimal places.	Column method with regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.
Multiplication	Recognising and making equal groups. Doubling Counting in multiples Use cubes, Numicon and other objects in the classroom	Arrays- showing commutative multiplication	Arrays 2d × 1d using base 10	Column multiplication- introduced with place value counters. (2 and 3 digit multiplied by 1 digit)	Column multiplication Abstract only but might need a repeat of year 4 first(up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication Abstract methods (multi-digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups? Use cubes and draw round 3 cubes at a time.	Division as grouping Division within arrays- linking to multiplication Repeated subtraction	Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction. 2d divided by 1d using base 10 or place value counters	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number including remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into the tenths and hundredths column too

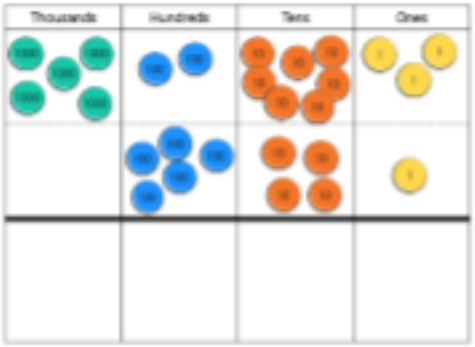
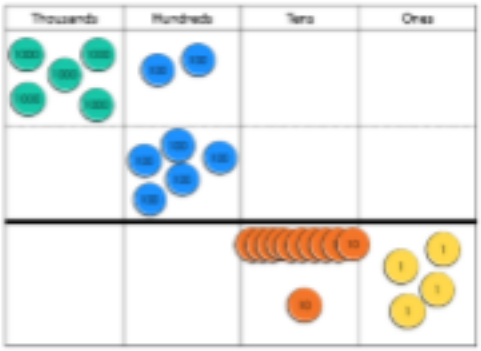
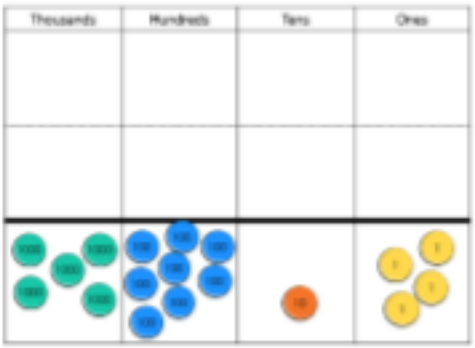
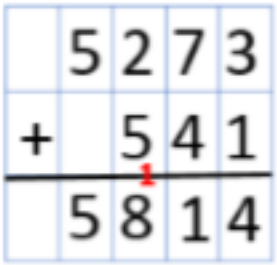
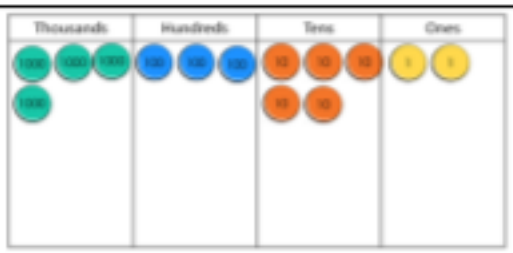
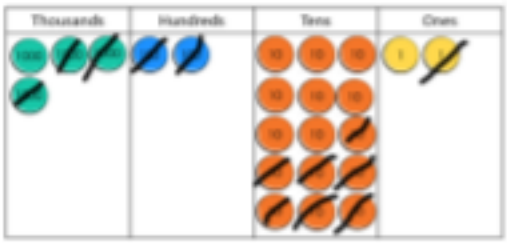
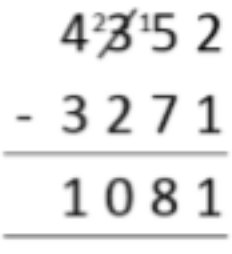
Progression in the teaching of place value

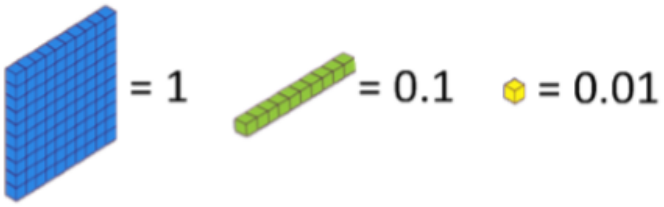
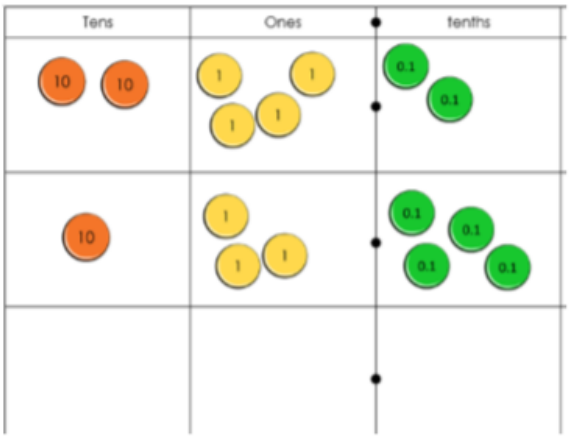
Foundation	Year 1	Year 2	Year 3 onwards
Understanding ten	Understanding numbers up to 20	Understanding numbers up to one hundred	Understanding numbers up to one thousand
A TENS FRAME is a simple maths tool that helps children: • Keep track of counting • See number relationships • Learn addition to 10 • Understand place value Use tens frames flash cards daily to ensure children recognise amounts. Use empty tens frames to fill with counters to enable children to understand number relationships. Either fill the tens frame in pairs or in rows. In rows shows 5 as a benchmark. Children can easily see more than 5 or less.  Setting the counters in pairs, naturally allows the children to see addition concepts. Include other visual images such as dice, cards, dominoes etc. 	'Ten' is the building block of our Base 10 numeration system. Young children can usually 'read' two-digit numbers long before they understand the effect the placement of each digit has on its numerical value. A child might be able to correctly read 62 as sixty two and 26 as twenty-six, and even know which number is larger, without understanding why the numbers are of differing values. Ten-frames can provide a first step into understanding two-digit numbers simply by the introduction of a second frame. Placing the second frame to the right of the first frame, and later introducing numeral cards, will further assist the development of place value understanding. 	Continue developing place value through the use of tens frames.  	Continue developing place value through the use of manipulatives including recognising 416 as 41 tens and 6 ones which is equivalent to 416 ones which is equivalent to four hundreds and one ten and six ones  Use Dienes blocks and place value charts 

Progression in the teaching of place value		
Year 4	**Year 5**	**Year 6**
Understanding numbers up to ten thousand	**Understanding numbers up to one million including decimals**	**Understanding numbers beyond one million including decimals**
Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters • Dienes blocks • Place value charts  Continue developing place value through the use of manipulatives including recognising the number above as one thousand plus two hundred plus four tens plus seven ones is equivalent to twelve hundred plus 47 ones etc. The children must also be able to identify that this number is also 12,470 tenths	Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters (including decimal counters) • Dienes blocks • Place value charts  They need to understand that there are no ten thousands in this number. The value of the digit 9 is nine thousand but there are three hundred and nine thousands in this number. They need to be able to recognise the value of the digit and the number and know that these are different. They also need to know how many tenths and hundredths are in this number 3092810 tenths and 30928100 hundredths in this number.	Continue developing place value through the use of manipulatives. • Place value arrow cards • Place value counters (including decimals counters) • Dienes blocks • Place value charts  They need to understand that there are no ten thousands in this number. The value of the digit 9 is nine thousand but there are 745309 thousands in this number. They need to be able to recognise the value of the digit and the number and know that these are different. They also need to know how many tenths, hundredths and thousandths there are in this number 7453092810 tenths and 74530928100 hundredths and 745309281000 thousandths in this number.

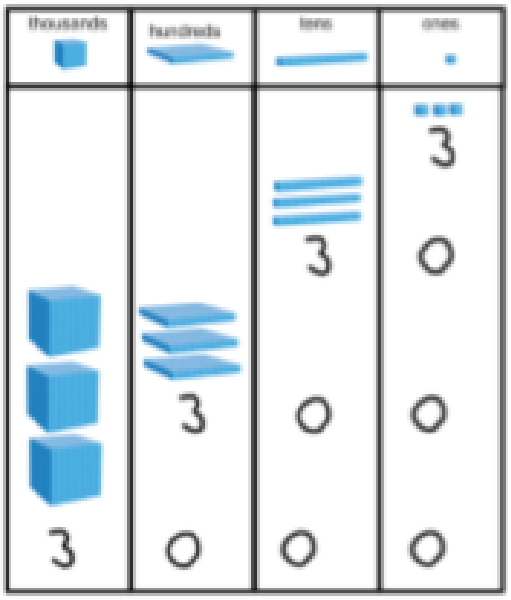
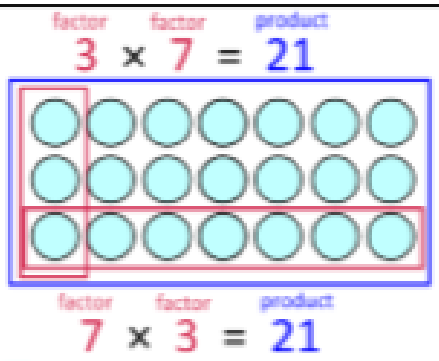
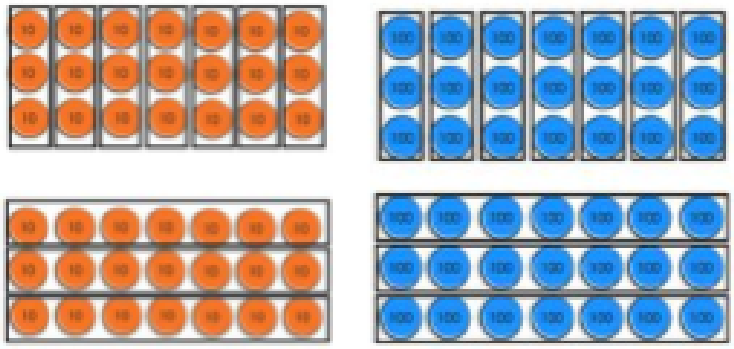
Y4 Addition & Subtraction

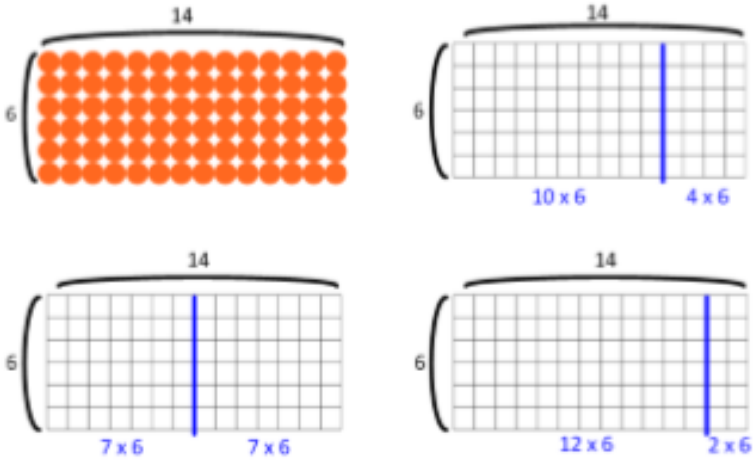
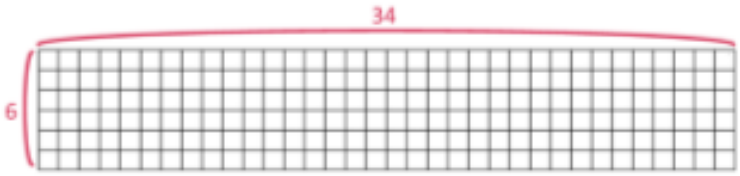
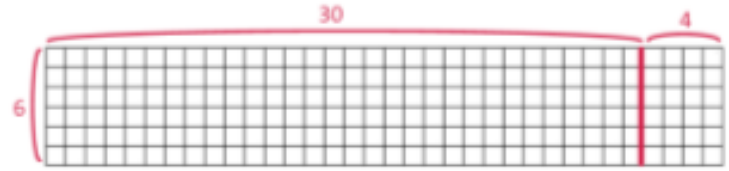
Strategies & Guidance	CPA
Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000. *Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.* **Count forwards and backwards in tenths and hundredths**	  Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes. E.g. 990 + 10 or 19.9 + 0.1
Using known facts and knowledge of place value to derive facts. **Add and subtract multiples of 10, 100 and 1000 mentally** *Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.*	
Adding and subtracting by partitioning one number and applying known facts. *By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.*	See Y3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery.

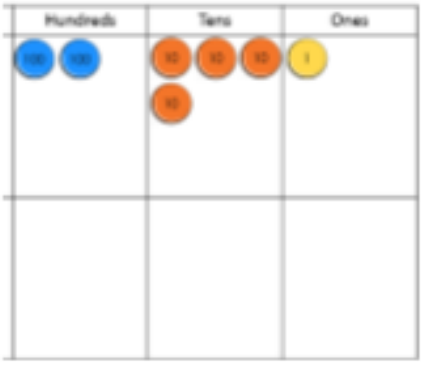
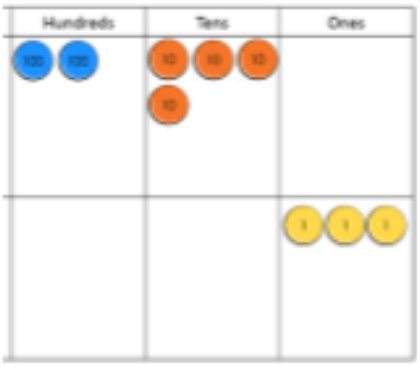
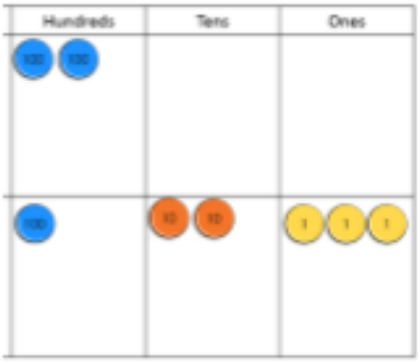
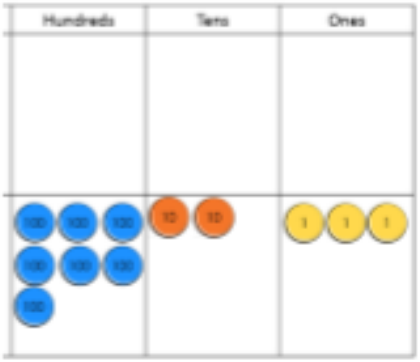
Strategies & Guidance	CPA
Written column methods for addition *Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.* *This method and the language to use are best understood through the tutorial videos found here on the toolkit.*	   
Written column methods for subtraction *Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.* *This method and the language to use are best understood through the tutorial videos on the toolkit.*	  

Strategies & Guidance	CPA
Calculating with decimal numbers *Assign different values to Dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these.* *Place value counters are another useful manipulative for representing decimal numbers.* *All of the calculation strategies for integers (whole numbers) can be used to calculate with decimal numbers.*	 24.2 + 13.4 = 

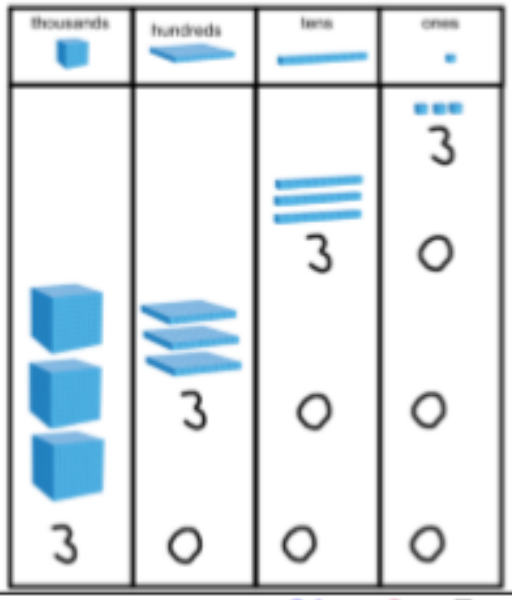
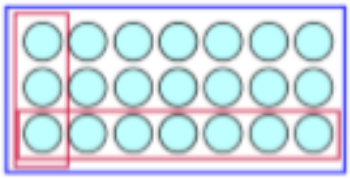
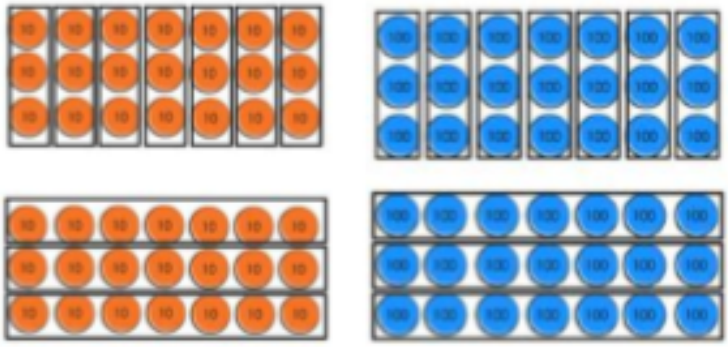
Y4 Multiplication

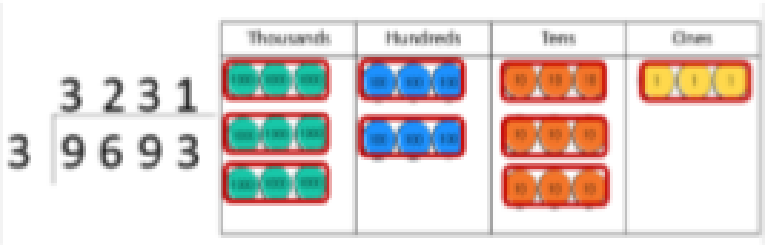
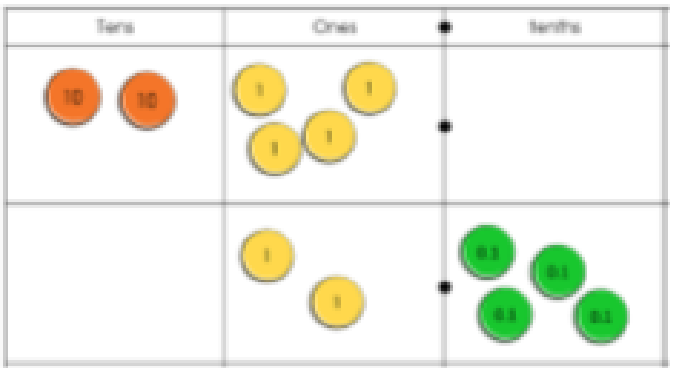
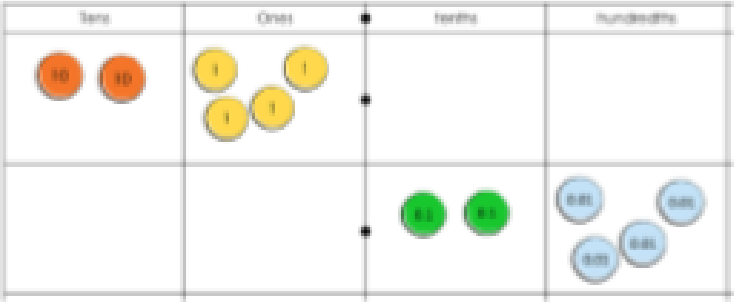
Strategies & Guidance	CPA
Multiplying by 10 and 100 *When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.* *When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.* *Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000*	 3 x 10 = 30 3 x 100 = 300 3 x 1000 = 3000
Using known facts and place value for mental multiplication involving multiples of 10 and 100 *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts.*	  30 x 7 = 210 70 x 3 = 210 7 x 30 = 210 3 x 70 = 210 300 x 7 = 2100 700 x 3 = 2100 7 x 300 = 2100 3 x 700 = 2100

Strategies & Guidance	CPA
Multiplying by partitioning one number and multiplying each part *Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations.* *As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.* **Distributive law** **a x (b + c) = a x b + a x c**	14 x 6  34 × 6  30 × 6 + 4 × 6 
Mental multiplication of three 1-digit numbers, using the associative law *Pupils first learn that multiplication can be performed in any order, before applying this to choose the most efficient order to complete calculations, based on their increasingly sophisticated number facts and place value knowledge.*	Four pots each containing two flowers which each have seven petals. How many petals in total?  (4 x 2) x 7 or 4 x (2 x 7)

Strategies & Guidance	CPA
Short multiplication of 3-digit number by 1-digit number *To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.* *With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation.* *This method and the language to use are best understood through the tutorial videos found here on the toolkit.*	Exemplification of this process is best understood through viewing the video tutorial     To calculate 241 x 3, represent the number 241. Multiply each part by 3, regrouping as needed. $$\begin{array}{r} 241 \\ \times \quad 3 \\ \hline 723 \\ \hline 1 \end{array}$$

Y4 Division

Strategies & Guidance	CPA
Dividing by 10 and 100 *When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.* *When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21*	 30 ÷ 10 = 3 300 ÷ 100 = 3 3000 ÷ 1000 = 3 300 ÷ 10 = 30 3000 ÷ 100 = 30 3000 ÷ 10 = 300
Derived facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	21 ÷ 3 = 7  21 ÷ 7 = 3  210 ÷ 7 = 30 210 ÷ 3 = 70 210 ÷ 30 = 7 210 ÷ 70 = 3 2100 ÷ 7 = 300 2100 ÷ 3 = 700 2100 ÷ 300 = 7 2100 ÷ 700 = 3

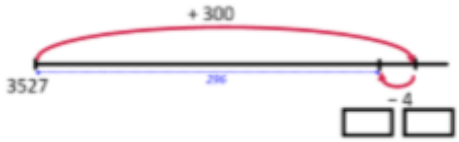
Strategies & Guidance	CPA
Short division of 4-digit numbers by 1-digit numbers *Pupils start with dividing 4-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used simultaneously in a place value chart, to develop conceptual understanding.* *They progress to calculations that require regrouping in the hundreds or tens columns.* *Pupils build on their conceptual knowledge of division to become confident with dividing numbers where the tens digit is smaller than the divisor, extending this to any digit being smaller than the divisor.* *Exemplification of this method and the language to use are best understood through viewing the tutorial videos found here on the toolkit.*	Division as sharing  Division as grouping 
Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths *When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.*	24 ÷ 10 = 2.4  24 ÷ 100 = 0.24 

Mental strategies for Maths Meetings:

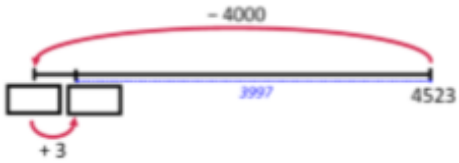
Round and adjust

Pupils should recognise that this strategy is useful when adding and subtracting near multiples of ten. They should apply their knowledge of rounding. It is very easy to be confused about how to adjust and so visual representations and mogical reasoning are essential to success with this strategy.

3527 + 296 = 3827 - 4



4523 – 3997 = 523 + 3



Near doubles

Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.

1600 + 1598 = double 1600 – 2